Distributed Data Aggregation for Sparse Recovery in Wireless Sensor Networks

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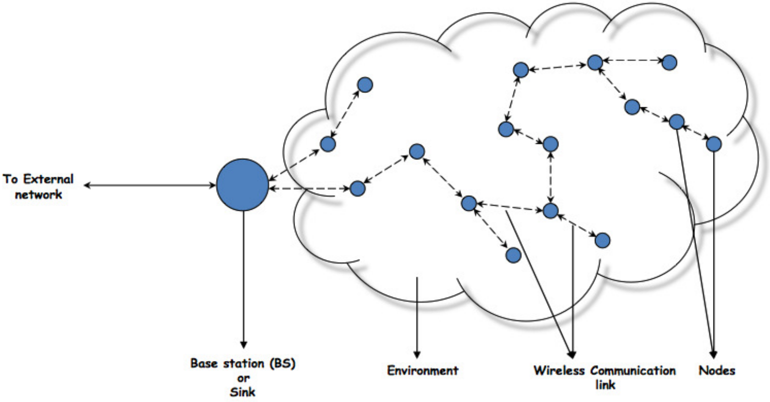
Outline

- Background and motivation
- Prior works
- Our approach
- Experiments
- Conclusion

Data aggregation in WSNs



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- i.e., average, mean of the data or transform the data in another domain (frequency, wavelet etc)
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- How to aquire informative data efficiently?

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- [Pros]: reduces the sample length
- [Cons]: introduces the dense measurement problem if Φ is dense. (i.e., a linear project would involve all the sensor readings

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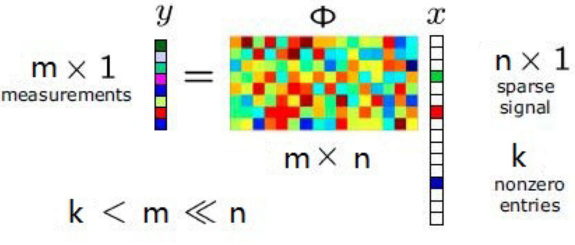

Figure : Compressed sensing [1]

[1] Image courtsey of Professor Richard Baraniuk at Rice University

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  - randomly choose m designated sensors
  - query only a necessary number of measurements (i.e., $\mathcal{O}(k\log(n))$ is enough for guaranteed CS data recovery)

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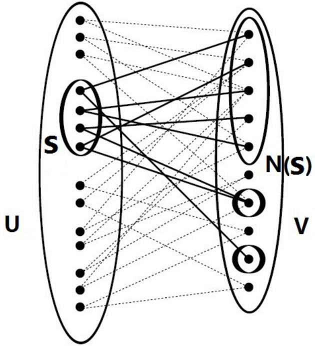
- [Berinde2008, Theorem 1,2,3] studied the relationship between the expander graph and CS measurement matrices, which serves as the theoretic foundation of our approach.

Our approach: measurement matrix design

Expander graph

- Let $X \subset U$, $N(X)$ be set of neighbors of X in V
- $G(U, V, E)$ is called (k, ϵ)-expander if
$$\forall X \subset U,\ |X| \leq k \Rightarrow |N(X)| \geq (1-\epsilon)d|X|$$
- Each set of nodes on the left expands to $N(X)$ number of nodes on right



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- Step 1: For each column, randomly generate τ integers whose values are between 1 and *m* and place 1's in those rows indexed by the τ numbers;
- Step 2: If the τ numbers in one column are not distinct, repeat Step 1 until they are (this is not really an issue when $\tau \ll m$).

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τ is the degree of the expander graph, $\tau = 8$ in our experiment

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Sparse binary matrix for sensor selection

For each row of the sparse binary matrix $\Phi_i \in \mathbb{R}^n$ $(1 \leq i \leq m)$

1	0	0	0	1	$\cdots$	0	1	0

can be seen as an *n*-dimensional row vector (binary indicator function) for sensor subset selection (i.e., select sensor j to be active for sensing when $\Phi_{ij} = 1$, and inactive otherwise.)

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- In the long run, the energy consumption can still be balanced

Our approach: DCSS algorithm

Network model

- Consider a wireless network of n sensors with diameter d hops, each measures a real data value x_i ($i = 1, 2, \cdots, n$), which is sparse or compressible under some transformation domains
- $y = \Phi x$ (Φ is sparse binary matrix, i.e., $\phi_{ij} \in \{0, 1\}$)
- Randomly choose m designated sensors from n sensors ($m \ll n$)

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  - $\Phi \in \mathbb{R}^{m \times n}$,
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  - The m designated sensors send their results to the FC and FC performs CS recovery for x^*.

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$$\begin{pmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{m1} & \phi_{m2} & \cdots & \phi_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ \cdots \\ y_m \end{pmatrix}$$

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For m designated sensors:

- Designated sensor D_i computes and stores the summation of the sensor reading it receives, for all $1 \leq i \leq m$
- Report $y_1, \cdots, y_m$ as observations $y \in \mathbb{R}^m$

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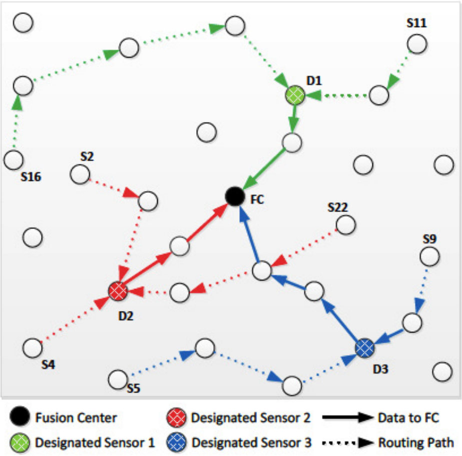
An example



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Communication cost

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- Assume d as the cost to send the projection to FC
- For generation of $\mathcal{O}(k \log(n))$ projection for data recovery, the total communication cost is: $\mathcal{O}(k(\tau\omega + d) \log(n))$

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Communication cost

TABLE I: Communication cost of different CS algorithms.

Algorithm	Cost
DS	$\mathcal{O}(kn \log n)$
SRP [9]	$\mathcal{O}(kd \log^2 n)$
CDS(RW) [4]	$\mathcal{O}(k(t+d) \log n)$
Sparse Binary	$\mathcal{O}(k(\tau\omega + d) \log n)$

- DS: dense sampling
- SRP: sparse random projection in [Wang2007IPSN]
- CDS (RW): compressive distributed sensing using random walk in [Sartipi2011DCC]

Experiments

Setup

- Data aggregation schemes comparsion
  - Dense sampling matrices (Random Gaussian, Scrambled Fourier measausrement matrices)
  - Sparse random projection matrices in [Wang2007IPSN] with various sparse level s
- Evaluation metrics

$$\varepsilon = \frac{\|\mathbf{x} - \mathbf{x}^*\|_2^2}{\|\mathbf{x}^*\|_2^2}$$

where $\mathbf{x}$ is the value of the original signal, while $\mathbf{x}^*$ is the reconstructed signal.

- Using ℓ_1-magic package for CS recovery

Experiments

Exact sparse signal recovery

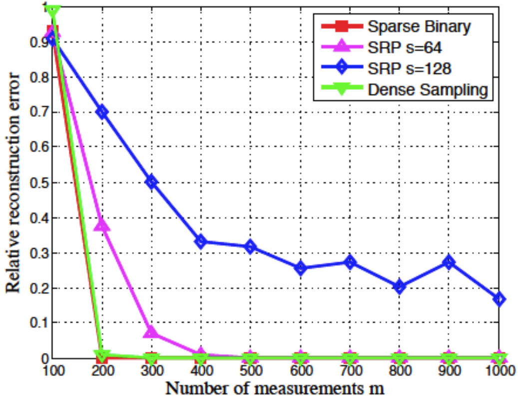

Fig. 5: Recovery result of an $n = 1024$, sparsity $k = 30$ sparse signal $\mathbf{x}$, with an average of 100 experiments using LP recovery method.

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Experiments

Noisy sparse signals recovery

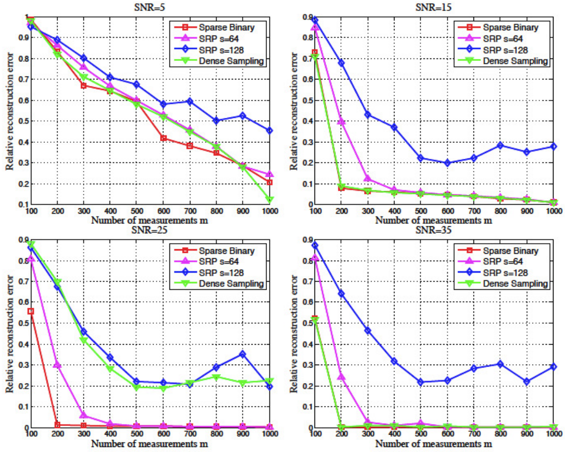

Fig. 6: Noisy recovery result of an $n = 1024$, sparsity $k = 30$ sparse signal $\mathbf{x}$, with different SNRs $(5, 15, 25,$ and $35)$ and an average of 100 experiments using LP recovery method evaluated by different measurements.

Experiments

Compressible signals recovery

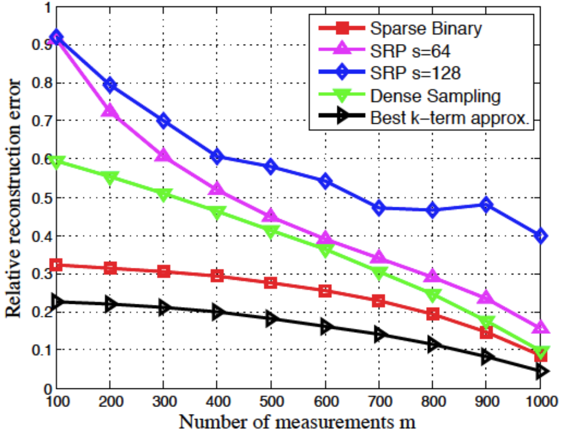

Fig. 7: Recovery result of a sampled compressible signal $x = 4n^{-\frac{7}{10}}$, with an average of 100 experiments using LP recovery method evaluated by different measurements.

Experiments

Real signal: Intel lab data (light intensity at node 19)

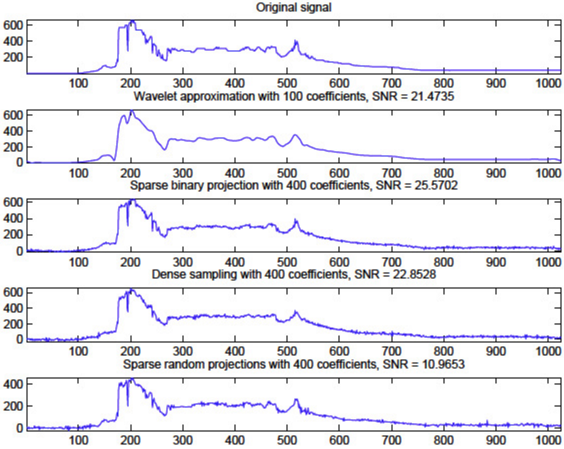

Fig. 8: Recovery result of real Intel lab signal using 100 wavelet coefficients and 400 CS measurements with different measurement matrices.

Experiments

Real signal: Intel lab data (light intensity at node 19)

TABLE II: Recovered SNR of different measurement matrices.

Methods	Wavelet approx.	Sparse binary	Dense sampling	SRP ($s = 64$)
SNR	21.4735	25.5702	22.8528	10.9653

Real signal: intel lab data (light intensity at node 19)

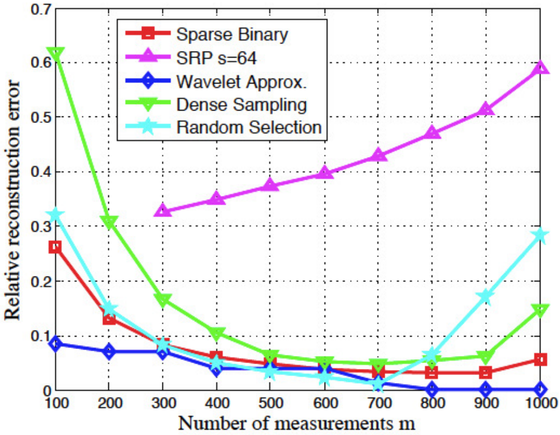

Fig. 9: Recovery result of real Intel lab signal, with an average of 100 experiments using LP recovery method evaluated by different measurements.

Conclusions

- A sparse binary measurement matrix was designed based on expaned graph.
  - Can be used for CS measurment matrix and sensor subset selection.
  - The recovery result is as good as traditional random dense CS measurement matrix and worked the best on compressible data.
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  - The recovery result is as good as traditional random dense CS measurement matrix and worked the best on compressible data.
  - Resolved the dense measurement problem.
- A structure free data aggregation algorithm (DCSS) was proposed. Results from both synthetic and real data experiments demonstrated the usefulness of the algorithms.

Key References

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Thank you!!